Integral as a "Net Change"

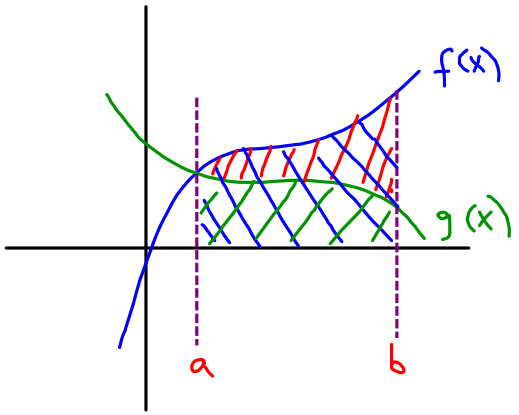
Look at problems from 7.1

$$\int_b^a f(x)\,dx = F(a) - F(b)$$

where $F(x)$ is an antiderivative of $f(x)$

Find $F(a)$

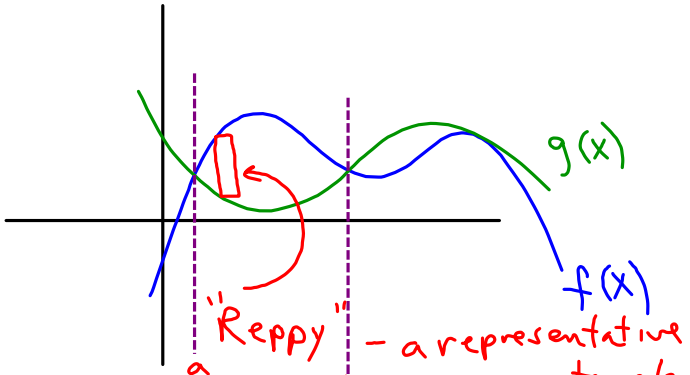
$$F(a) = F(b) + \int_b^a f(x)\,dx$$



$$\text{Area} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

or

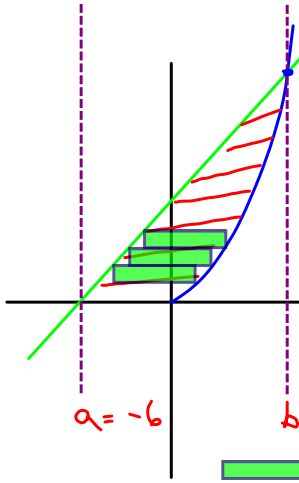
$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$



"Reppy" - a representative rectangle

Reppy's height $= f(x) - g(x)$

$$\text{Area} = \int_a^b f(x) - g(x)\,dx$$



$x = g(y)$

$x = f(y)$

$x = y - 6$

$x = \sqrt{y}$

$h = \sqrt{y} - (y-6)$

$$\text{Area} = \int_0^9 \sqrt{y} - y + 6 \, dy$$

$$= \frac{2}{3}y^{\frac{3}{2}} - \frac{y^2}{2} + 6y \Big|_0^9$$

$$= \frac{2}{3}\left(9^{\frac{3}{2}}\right) - \frac{9^2}{2} + 6(9)$$

$$= 18 - \frac{81}{2} + 54$$

$$= \frac{144}{2} - \frac{81}{2}$$

$$= \frac{63}{2}$$