$$\frac{dy}{dx} = 2x(1+y^2)e^{x^2}$$

$$\int \frac{1}{1+y^2}\frac{dy}{dx}\,dx = \int 2xe^{x^2}\,dx$$

$$\int \frac{1}{1+y^2}\,dy = \int 2xe^{x^2}\,dx$$

Let $u = x^2$

$du = 2x\,dx$

(1) Do you see a function in a function?

(2) Do you see its derivative hanging around?

$$\tan^{-1} y + C = \int e^u\,du$$

$u = x^2$

$$\frac{du}{dx} = 2x$$

$du = 2x\,dx$

$$\tan^{-1} y = e^{x^2} + C$$

$$y = \tan(e^{x^2} + C)$$

$$\int 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3 + 1$

$du = 3x^2\,dx$

$$\int \sqrt{u}\,du$$

$$\frac{2}{3}u^{\frac{3}{2}} + C$$

$$\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} + C$$

$$\frac{2\sqrt{x^3+1}^3}{3} + C$$

$$\frac{2(x^3+1)\sqrt{x^3+1}}{3} + C$$

$$\frac{(2x^3+2)\sqrt{x^3+1}}{3} + C$$

$$\frac{1}{3}\int 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3+1$

$du = 3x^2\,dx$

$$\frac{1}{3}\int \sqrt{u}\,du$$

$$\frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$$

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}} + C$$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3 + 1$

$du = 3x^2 dx$

$u = (-1)^3 + 1$

$u = 0$

$u = 1^3 + 1$

$u = 2$

$$\int_0^2 \sqrt{u}\,du$$

$$\frac{2}{3}2^{\frac{3}{2}} - \frac{2}{3}0^{\frac{3}{2}}$$

$$\frac{2\sqrt{8}}{3}$$

$$\frac{4\sqrt{2}}{3}$$

$\sqrt{8} = \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

$$\int \sqrt{\tan x}\, \sec^2 x\, dx$$

Let $u = \tan x$

$du = \sec^2 x\, dx$

$$\int \sqrt{u}\, du$$

$$\frac{2}{3} u^{\frac{3}{2}} + C$$

$$\frac{2}{3} \sqrt{\tan x}^{3} + C$$