Warm-up

① $\frac{1}{3}\int 3\cdot 4x\, e^{6x^2+2}\, dx$

$u = 6x^2+2$

$du = 12x\, dx$

$= \frac{1}{3}\int e^u\, du$

$\boxed{= \frac{1}{3} e^{6x^2+2} + C}$

② $\int x e^{3x}\, dx$

+	x	e^{3x}
−	1	$\frac{1}{3}e^{3x}$
	0	$\frac{1}{9}e^{3x}$

$\boxed{= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C}$

③ $\int \frac{1}{(x-4)(x+3)}\, dx$

$= \frac{1}{7}\int \frac{1}{x-4} - \frac{1}{x+3}\, dx$

$= \frac{1}{7}\left(\ln|x-4| - \ln|x+3|\right) + C.$

Review:

$$\frac{1}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

$$\frac{1}{(x-4)(x+3)} = \frac{A(x+3) + B(x-4)}{(x-4)(x+3)}$$

$$1 = A(x+3) + B(x-4)$$

$$1 = Ax + 3A + Bx - 4B$$

How many x's ??

x-terms

So $\boxed{\begin{aligned} A + B &= 0 \\ 3A - 4B &= 1 \end{aligned}}$

and

$$\begin{aligned} 4A + 4B &= 0 \\ 3A - 4B &= 1 \\ \hline 7A &= 1 \end{aligned}$$

$$A = 1/7$$

$$B = -1/7$$

ex $\displaystyle\int \frac{1}{(x+5)(x+1)}\,dx$

$$= \int \frac{-1/4}{x+5} + \frac{1/4}{x+1}\,dx$$

$$= \frac{1}{4}\int \frac{-1}{x+5} + \frac{1}{x+1}\,dx$$

$$= -\frac{1}{4}\ln|x+5| + \frac{1}{4}\ln|x+1| + C$$

$$\frac{A}{x+5} + \frac{B}{x+1} = \frac{1}{(x+5)(x+1)}$$

$$Ax + A + Bx + 5B = 1$$

$$Ax + Bx + A + 5B = 1$$

$$\begin{cases} A + B = 0 \\ A + 5B = 1 \end{cases}$$

$$\boxed{\begin{aligned} A &= -1/4 \\ B &= 1/4 \end{aligned}}$$

Integration by Parts (cont.)

ex $\int e^x \sin x\,dx$

$u = \sin x \qquad v = e^x$

$du = \cos x\,dx \qquad dv = e^x\,dx$

$= e^x \sin x - \int e^x \cos x\,dx$

$u = \cos x \qquad v = e^x$

$du = -\sin x\,dx \qquad dv = e^x\,dx$

$= e^x \sin x - \left(e^x \cos x + \int e^x \sin x\,dx\right)$

$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$

STOP and Recap.

$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$

like

$u = \text{stuff} - u$

solve for u.

$2u = \text{stuff}$

$u = \text{stuff}/2$

$$\boxed{\int e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2} + C}$$

Try $\int e^{2x} \sin 4x \, dx$

for "fun".

ex

$$\int x^2 \ln x \, dx$$

$u = x^2 \qquad v = ??$

$\downarrow \qquad\qquad \uparrow$

$du = 2x\,dx \qquad dv = \ln x\,dx$

$u = \ln x \qquad v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx \qquad dv = x^2 dx$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$\boxed{= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

ex $\int \ln x \, dx$

$u = \ln x$ $\quad$ $v = x$

$du = \frac{1}{x} dx$ $\quad$ $dv = dx$

$= x \cdot \ln x - \int x \cdot \frac{1}{x} dx$

$= x \cdot \ln x - \int 1 \, dx$

$\boxed{= x \ln x - x + C}$