Integration by Substitution (cont.)

ex $\int \frac{16x}{(4x^2+3)^3}\,dx$

$u = 4x^2+3$

$\frac{du}{dx} = 8x$

$du = 8x\,dx$

$= \int \frac{2 \cdot 8x}{(4x^2+3)^3}\,dx$

$= 2\int \frac{1}{u^3}\,du$

$= 2\int u^{-3}\,du$

$= \frac{2}{-2} \cdot u^{-2} + C$

$\boxed{= -1u^{-2} + C}$

$= \frac{-1}{(4x^2+3)^2} + C$

$$= \frac{-1}{(4x^2+3)^2} + C$$

Check:

$$\frac{d}{dx}\left[\frac{-1}{(4x^2+3)^2} + C\right]$$

$$= \frac{d}{dx}\left[-1(4x^2+3)^{-2} + C\right]$$

$$= 2(4x^2+3)^{-3} \cdot 8x + 0$$

$$= \frac{16x}{(4x^2+3)^3}$$

ex $2\int \frac{1}{2}\tan^7(x/2)\sec^2(x/2)\,dx$

Choices for u:

$u = x/2$

$\boxed{u = \tan(x/2)}$

$u = \sec(x/2)$

$du = \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}\,dx$

$= 2\int u^7\,du$

$= 2\cdot\frac{1}{8}u^8 + C$

$$\boxed{= \frac{1}{4}(\tan(x/2))^8 + C}$$

ex $\int_{-4}^{4} x\sqrt{16-x^2}\,dx$

OPTION #1 : $F(4) - F(-4)$

$F(x) = \int x\sqrt{16-x^2}\,dx$

That is, do the bounds last.

OPTION 2: I ♡ u

"I'm going to u-land"

$\int_{-4}^{4} x\sqrt{16-x^2}\,dx$

$u = 16 - x^2$

$du = -2x\,dx$

$= -\frac{1}{2}\int_{x=-4}^{x=4} -2x\sqrt{16-x^2}\,dx$

If $x=4$, $u=16-4^2$

$u=0$

If $x=-4$, $u=0$

$= -\frac{1}{2}\int_{u=0}^{u=0} \sqrt{u}\,du$

$\boxed{= 0}$

ex $-\frac{1}{2}\int_0^5 -2x\sqrt{25-x^2}\,dx$

$u = 25 - x^2$

$du = -2x\,dx$

If $x=0$

$u = 25 - 0^2$

$u = 25$

$= -\frac{1}{2}\int_{25}^{0} \sqrt{u}\,du$

$= \frac{1}{2}\int_0^{25} u^{1/2}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^{25}$

$= \frac{1}{3}25^{3/2} - \frac{1}{3}0^{3/2}$

$= \frac{125}{3}$

Careful!!!

$$\int_0^5 x\sqrt{25-x^2}\,dx \qquad u = 25 - x^2$$

$$du = -2x\,dx$$

$$= -\frac{1}{2}\int_0^5 \sqrt{u}\,du$$ WRONG!!

$$= -\frac{1}{2}\,\frac{2}{3}\,u^{3/2}\Big|_0^5$$

$$= -\frac{1}{3}\left(25-x^2\right)^{3/2}\Big|_0^5$$ WRONG!

$$= -\frac{1}{3}\left(0^{3/2}\right) + \frac{1}{3}(25)^{3/2}$$

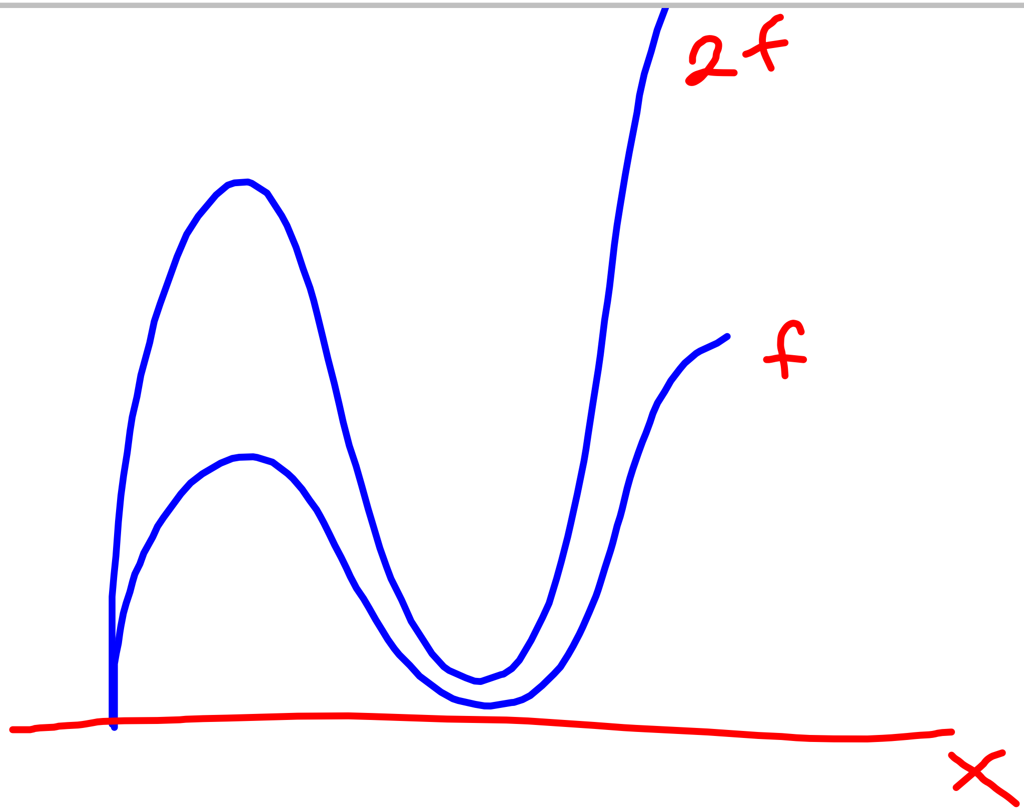
$$\int_{43}^{1984} \underbrace{f(x)\,dx}_{??\,du}$$

$$u = (x-1984)(x-43)$$

NO!!

NO!!

ex $\int \cos x \, dx$


2f
f
x